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Chaotic Synchronization in Synaptically Coupled BVP Neurons with External Impulsive Forces

Hiroyuki Kitajima * Kunichika Tsumoto † Hiroshi Kawakami ‡
Tetsuo Hattori *

Abstract — **We investigate synchronous chaotic firing observed in synaptically coupled BVP neurons with external impulsive forces. The effect of changing time delays for both inhibitory and excitatory coupling neurons is studied. We show that only excitatory coupling neurons produce synchronous chaotic firing patterns for a small delay. By increasing a delay both two types of coupling neurons generate asynchronous chaotic firing patterns.**

1 INTRODUCTION

Synchronization of neurons has attracted much interest in relation to information processing in the brain. It is well known that some neurons show chaotic or irregular activities, thus chaotic synchronization is one of the most important topics to understand neural networks (see [1–3] and references therein). Considering a system of coupled neurons the effect of a time delay have become a subject of intense research activities [4–6]. Vreeswijk *et al.* [7] studied the effect of changing a time delay and showed that inhibitory connection synchronizes periodic neural firing for a large delay.

In this paper we examine synchronization of chaotic neural firing observed in synaptically coupled Bonhöffer van der Pol (BVP or FitzHugh-Nagumo) neurons with external impulsive forces. The effect of changing time delays for both inhibitory and excitatory coupling neurons is studied. We show that only excitatory coupling neurons produce synchronous chaotic firing patterns for a small delay. By increasing a delay both two types of coupling neurons generate asynchronous chaotic firing patterns.

*Department of Reliability-based Information Systems Engineering, Kagawa University, 2217-20 Hayashi, Takamatsu, 761-0397 Japan, e-mail: [kitaji, hattori]@eng.kagawa-u.ac.jp, tel.: +81 87 8642226, fax: +81 87 8642262.

†Department of Telecommunications, Takuma National College of Technology, 551 Koda, Takuma, Mitoyo, Kagawa 769-1103 Japan, e-mail: tsumoto@dt.takuma-ct.ac.jp.

‡Department of Electrical and Electronic Engineering, Tokushima University, 2-1 Minami-Josanjima, Tokushima, 770-8506 Japan, e-mail: kawakami@ee.tokushima-u.ac.jp.

2 SYSTEM EQUATION

2.1 BVP Neuron with Impulse

The BVP or FitzHugh-Nagumo equation is a well-known neuron model representing the electrical behavior across a nerve membrane and has been widely studied [8–12]. The equation of the single BVP neuron with the external impulsive force is described as

$$\frac{dx_i}{dt} = c\left(x_i - \frac{1}{3}x_i^3 + y_i\right) + h\sum_{k\in Z}\delta(t - 2k\pi/\omega)$$
$$\frac{dy_i}{dt} = -\frac{1}{c}(x_i + by_i + a) \qquad (i = 1,\ 2) \tag{1}$$

where $\delta(t)$ is the Dirac's delta function and, h and ω are the amplitude and the angular frequency of the impulsive force, respectively. The system parameters are fixed as

$$a = 0.7,\ b = 0.8,\ c = 3.0 \tag{2}$$

for the occurrence of a stable equilibrium point in the system of Eq. (1).

2.2 Coupled BVP Neurons with Impulse

The equation of synaptically coupled BVP neurons is written as

$$\frac{dx_i}{dt} = c\left(x_i - \frac{1}{3}x_i^3 + y_i + z_i\right) + h\sum_{k\in Z}\delta(t - 2k\pi/\omega)$$
$$\frac{dy_i}{dt} = -\frac{1}{c}(x_i + by_i + a) \qquad (i = 1,\ 2) \tag{3}$$

where the coupling term z_i is

$$z_i = -d(x_i - \hat{x})\alpha_{i+1} \qquad (\alpha_3 \equiv \alpha_1) \tag{4}$$

and α_i is given as the solution of these equations

$$\frac{d\alpha_i}{dt} = \frac{\beta_i}{\tau}$$
$$\frac{d\beta_i}{dt} = -2\,\frac{\beta_i}{\tau} - \frac{\alpha_i}{\tau}. \tag{5}$$

Note that the solution $\alpha_i(t)$ of Eq. (5) with initial condition $(\alpha_i,\ \beta_i) = (0,\ 1)$ at $t = 0$ is calculated as $\alpha_i(t) = (t/\tau)\exp^{-t/\tau}$ called α-function (see [8, 13] and the references cited therein). In Eq. (4) $\hat{x}$ represents the synaptic reversal potential, which depends on the type of synaptic transmitter released from a presynaptic neuron and their receptors. The coupling becomes excitatory and inhibitory with $\hat{x} > x_{\text{eq}}$ and $\hat{x} < x_{\text{eq}}$, respectively, where x_{eq} denotes an equilibrium potential of the single BVP neuron ($x_{\text{eq}} \simeq -1.2$ for parameter setting in Eq. (2)) .

We assume that a firing of the membrane potential of the BVP neuron occurs when the state variables x_i crosses zero as a threshold value, changing its sign from negative to positive. Each vector (α_i, β_i) jumps to the constant (0, 1) at $t = t_0 + \tau_d$ where t_0 is the time when x_i changes to $x_i > 0$. Namely, the firing information of a neuron transforms to the other neuron with the time delay τ_d.

3 NUMERICAL RESULTS

The values of parameters of synaptic characteristics are fixed as

$$\tau = 2.0, \ d = 1.0. \tag{6}$$

Figure 1 shows waveforms of chaotic solutions observed in Eq. (3) with $\hat{x} = -0.3$ (excitatory coupling) and $\hat{x} = -1.5$ (inhibitory coupling). In Fig. 2 we show approximate firing rates $r(t)$ of Fig. 1 calculated using the following equation [14]

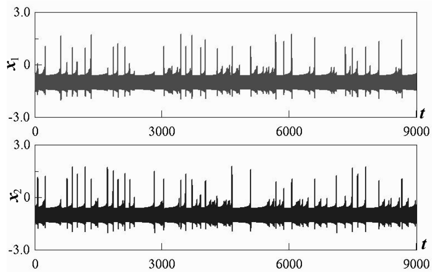
$$r(t) = \sum_{j=1}^{n} w(t - t_j) \qquad (j = 1, \ 2, \ \cdots, n) \tag{7}$$

where t_j is the time when the n spikes occurred, and $w(t)$ is a Gaussian window function described by

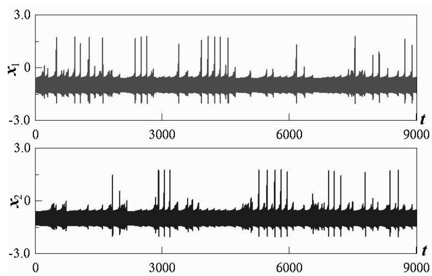
$$w(t) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{t^2}{2\sigma^2}\right) \tag{8}$$

with $\sigma = 200$. From Fig. 2 we can see that the chaotic firing trains of excitatory coupling neurons are synchronized at in-phase, while inhibitory coupling neurons produce almost anti-phase synchronized states.

The bifurcation mechanism of generation of such chaotic firing is shown in Fig. 3. When the value of the amplitude h of impulsive forces is less than 0.6145, two neurons never fire, thus neurons are not connected synaptically. At the point marked by ①, the saddle-node bifurcation of a one-periodic point occur and stable two-periodic points appear. This two-periodic points meet successive period-doubling bifurcations around $h = 0.612$ marked by ② and chaotic solution is generated. Increasing the value of h, two neurons start to fire and be synaptically connected each other at the point marked by ③.



(a) Excitatory coupling. $\hat{x} = -0.3$.



(b) Inhibitory coupling. $\hat{x} = -1.5$.

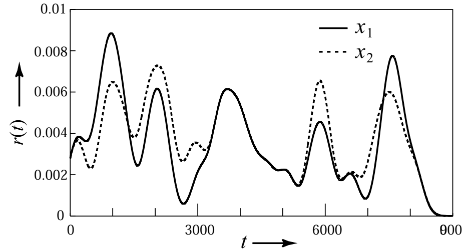
Figure 1: Waveforms of chaotic oscillations in Eq. (3) with $\omega = 1.5$, $h = 0.6148$ and $\tau_d = 1.5$.

Next we show the results of increasing the time delay τ_d for both of inhibitory and excitatory coupling system. Figure 4 and 5 represent, respectively, the waveforms and their firing rates in Eq. (3) with $\tau_d = 3.5$. In the excitatory coupling system, when the time delay is small, two neurons produce synchronous firing. However, by increasing the time delay synchronization of firing pattern of two neurons is broken. On the other hand in inhibitory coupling system, asynchronous firing pattern is kept against increasing the time delay.

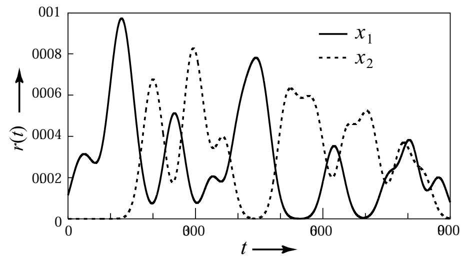
We define that if the time difference of firing rate's peak of two neurons are within 100 then two neurons produce synchronous firing. In Fig. 6 we show the results of counting synchronous firing for both excitatory and inhibitory coupling system by changing the values of the time delay. Inhibitory coupling neurons always generate asynchronous chaotic firing pattern for increasing the time delay. On the other hand synchronous chaotic firing pattern of excitatory coupling neurons becomes asynchronous over delay = 2.5.

4 CONCLUSIONS

In this paper we have examined synchronization of chaotic neural firing observed in synaptically cou-



(a) Firing rates of Fig. 1(a). (excitatory coupling)



(b) Firing rates of Fig. 1(b). (inhibitory coupling)

Figure 2: Firing rates. $\tau_d = 1.5$.

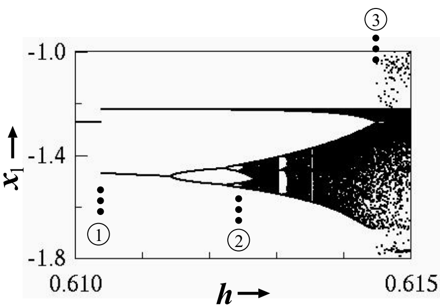
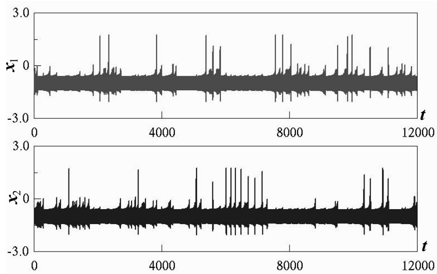
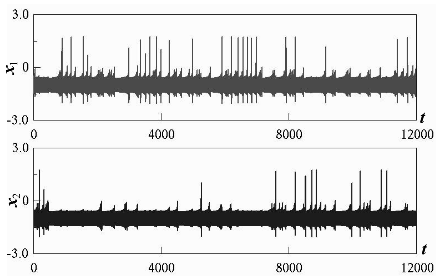


Figure 3: One-parameter bifurcation diagram of excitatory coupling neurons. $\omega = 1.5$.

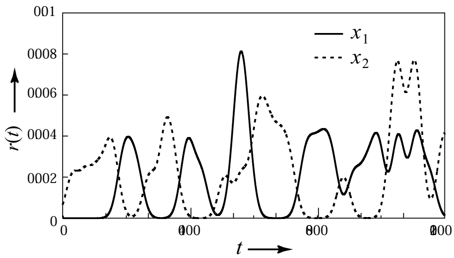


(a) Excitatory coupling. $\hat{x} = -0.3$.

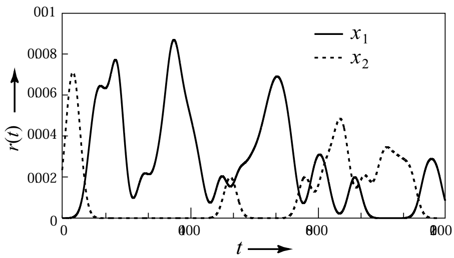


(b) Inhibitory coupling. $\hat{x} = -1.5$.

Figure 4: Waveforms of chaotic oscillations in Eq. (3) with $\omega = 1.5$, $h = 0.6148$ and $\tau_d = 3.5$.



(a) Firing rates of Fig. 4(a). (excitatory coupling)



(b) Firing rates of Fig. 4(b). (inhibitory coupling)

Figure 5: Firing rates. $\tau_d = 3.5$.

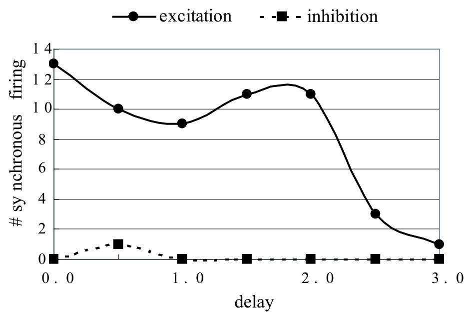


Figure 6: Number of synchronous firing.

pled Bonhöffer van der Pol (BVP or FitzHugh-Nagumo) neurons with external impulsive forces. The effect of changing time delays for both inhibitory and excitatory coupling neurons is studied. We showed that only excitatory coupling neurons produce synchronous chaotic firing patterns for a small delay. By increasing a delay both two types of coupling neurons generate asynchronous chaotic firing patterns.

This result is different from the periodic firing case [7]. Thus, it is interesting open problems to study the mechanisms of generating synchronous chaotic firing.

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